

Edexcel GCE

Mathematics

Mechanics M6 6682

Summer 2005

publication

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Confidential Mark Scheme

Edexcel GCE

Mathematics

N 23738 A



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**EDEXCEL**

**GENERAL CERTIFICATE OF EDUCATION**

**Advanced Subsidiary/Advanced Level**

**Mechanics M6**

**MARKING SCHEME**

**June 2005**

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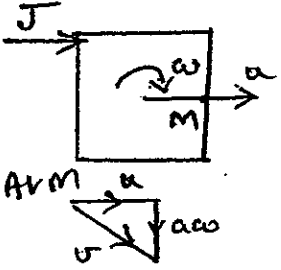
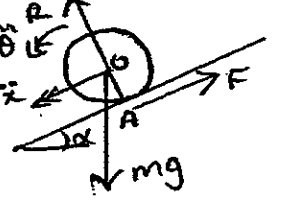
**Tel.: 01491 659091**

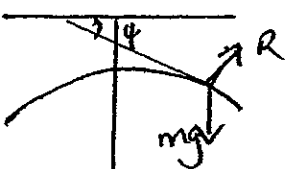
**Marking should be completed by 18 July 2005.**

## General Instructions

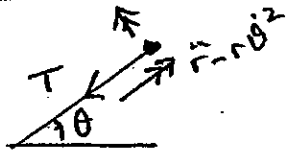
1. The total number of marks for the paper is 75.
2. Method (M) marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
3. Accuracy (A) marks can only be awarded if the relevant method (M) marks have been earned.
4. (B) marks are independent of method marks.
5. Method marks should not be subdivided.
6. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. Indicate this action by 'MR' in the body of the script (but see also note 10).
7. If a candidate makes more than one attempt at any question:
  - (a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - (b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
8. Marks for each question, or part of a question, must appear in the right-hand margin and, in addition, total marks for each question, even where zero, must be ringed and appear in the right-hand margin and on the grid on the front of the answer book. It is important that a check is made to ensure that the totals in the right-hand margin of the ringed marks and of the unringed marks are equal. The total mark for the paper must be put on the top right-hand corner of the front cover of the answer book.
9. For methods of solution not in the mark scheme, allocate the available M and A marks in as closely equivalent a way as possible, and indicate this by the letters 'OS' (outside scheme) put alongside in the body of the script.
10. All A marks are 'correct answer only' (c.a.o.) unless shown, for example, as A1 f.t. to indicate that previous wrong working is to be followed through. In the body of the script the symbol  $\checkmark$  should be used for correct f.t. and  $\times$  for incorrect f.t. After a misread, however, the subsequent A marks affected are treated as A f.t., but manifestly absurd answers should never be awarded A marks.
11. Ignore wrong working or incorrect statements following a correct answer.

June 2005  
6682 Mechanics M6  
Mark Scheme

Question Number	Scheme	Marks
1.	 <p> <math>\text{Lin. mom}^m : J = 4mu</math>  <math>\text{Ang mom}^m : Ja = \frac{16ma^2 \cdot \omega}{3}</math>  <math>v^2 = u^2 + (a\omega)^2</math>  <math>v = \sqrt{\left(\frac{J}{4m}\right)^2 + \left(\frac{3J}{16m}\right)^2}</math>  <math>= \frac{5J}{16m}</math> </p>	<p>m1 A1 m1 A1 m1 A1 ↓ m1 A1 (8)</p>
2.	 <p> <math>A \vee A \quad \dot{x} = a\dot{\theta} \Rightarrow \ddot{x} = a\ddot{\theta}</math>  <math>R(\nearrow) \quad R = mg \cos \alpha</math>  <math>R(\nearrow) \quad m\ddot{x} = mg \sin \alpha - F \quad \text{--- (1)}</math>  <math>M(\circ) \quad Fa = \frac{2}{3}ma^2\ddot{\theta}</math>  <math>\text{Eliminate } \ddot{x} \text{ \&amp; } \ddot{\theta} : mg \sin \alpha - F = \frac{3F}{2}</math>  <math>\Rightarrow F = \frac{2mg \sin \alpha}{5}</math>  <math>F \leq \mu R \Rightarrow \frac{2mg \sin \alpha}{5} \leq \mu mg \cos \alpha</math>  <math>\Rightarrow \underline{\mu \geq \frac{2}{5} \tan \alpha} \quad \text{--- (*)}</math>  <math>\text{From (1)} \quad m\ddot{x} = mg \sin \alpha - \frac{2}{5}mg \sin \alpha</math>  <math>\Rightarrow \underline{\ddot{x} = \frac{3}{5}g \sin \alpha}</math> </p>	<p>B1 B1 m1 A1 { m1 A1 + m1 A1 m1 A1 A1 A1 (12)</p>

Question Number	Scheme	
3.	<p>(a) Impulse from wall is <math>\leftarrow</math> &amp; does not affect vert<sup>l</sup> motion</p> <p>(<math>\uparrow</math>): <math>0 = U \sin \alpha t - \frac{1}{2} g t^2</math></p> <p><math>t = (0 \text{ or } ) \frac{2U \sin \alpha}{g} \quad (*)</math> <span style="float: right;">CSO</span></p> <p>(b) At wall (<math>\Leftrightarrow</math>) speed of rebound <math>\neq e u \cos \alpha</math></p> <p>Time from O to wall = <math>\frac{d}{U \cos \alpha}</math></p> <p>Time from wall to O = <math>\frac{d}{e U \cos \alpha}</math></p> <p><math>\Rightarrow \frac{2U \sin \alpha}{g} = \frac{d}{U \cos \alpha} + \frac{d}{e U \cos \alpha}</math></p> <p><math>\Rightarrow 2U^2 \sin \alpha \cos \alpha = g d \left(1 + \frac{1}{e}\right)</math></p> <p><math>\Rightarrow \underline{U^2 \sin 2\alpha = g d \left(1 + \frac{1}{e}\right)} \quad (*)</math></p>	<p>M1</p> <p>M1 A1</p> <p>A1 (4)</p> <p>M1 A1</p> <p>B1</p> <p>B1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (8)</p> <p>(12)</p>
4.	 <p><math>R(\leftarrow) \quad mg \cos \psi - R = \frac{m v^2}{\rho} \quad \text{--- (1)}</math></p> <p><math>S = a \tan \psi \Rightarrow \rho = \frac{ds}{d\psi} = a \sec^2 \psi</math></p> <p><math>\frac{dy}{d\psi} = \sin \psi \Rightarrow dy = \sin \psi \frac{ds}{d\psi} d\psi</math></p> <p><math>y = a \int \sin \psi \sec^2 \psi d\psi</math></p> <p><math>= a \int \sec \psi \tan \psi d\psi = a \sec \psi + C</math></p> <p><math>\psi = 0, y = a \Rightarrow C = 0</math></p> <p>Energy: <math>\frac{1}{2} m v^2 = mg(y - a)</math></p> <p><math>\Rightarrow v^2 = 2ga(\sec \psi - 1)</math></p> <p>Put <math>R=0</math> in (1) &amp; sub for <math>v^2</math> &amp; <math>\rho</math>:</p> <p><math>mg \cos \psi = \frac{m \cdot 2ga(\sec \psi - 1)}{a \sec^2 \psi}</math></p> <p><math>\Rightarrow \cos \psi = 2(\cos \psi - \cos^2 \psi)</math></p> <p><math>\cos \psi \neq 0 \Rightarrow \cos \psi = \frac{1}{2} \Rightarrow \psi = \frac{\pi}{3} \quad (*)</math></p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p> </p> <p>M1 A1</p> <p> </p> <p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>A1</p> <p>(12)</p>

5.  
(a)



$$R(\curvearrowright) \quad 0 = 2 \cdot \frac{1}{r} \frac{d}{dt} (r^2 \dot{\theta}) \Rightarrow r^2 \dot{\theta} = h \quad (\text{const})$$

M1

$$t=0, r=5, \dot{r}=1, \omega = \frac{3}{5}, r\dot{\theta} = 1 \cdot \sin \alpha = \frac{4}{5}$$

M1

$$\Rightarrow h = 5 \cdot \frac{4}{5} = \underline{4} \quad (*)$$

A1 (4)

(b)

$$R(\curvearrowright) \quad -\frac{10}{r^2} = 2(\ddot{r} - r\dot{\theta}^2)$$

M1 A1

$$-\frac{10}{r^2} = 2(\ddot{r} - r \cdot \left(\frac{4}{r^2}\right)^2)$$

M1

$$\ddot{r} = -\frac{5}{r^2} + \frac{16}{r^3}$$

$$\int \dot{r} d\dot{r} = \int -\frac{5}{r^2} + \frac{16}{r^3} dr$$

M1

$$\frac{1}{2} \dot{r}^2 = \frac{5}{r} - \frac{8}{r^2} + C$$

A1

$$t=0, r=5, \dot{r}=\frac{3}{5} : \frac{9}{50} = \frac{5}{5} - \frac{8}{25} + C \Rightarrow C = -\frac{1}{2}$$

M1 A1

moving  
P/L  $\perp$   $\vec{F}$  when  $\dot{r}=0 \Rightarrow \frac{5}{r} - \frac{8}{r^2} - \frac{1}{2} = 0$

M1

$$\Rightarrow r^2 - 10r + 16 = 0$$

M1

$$(r-2)(r-8) = 0$$

$$\underline{r = 2 \text{ or } 8}$$

A1 (10)

(14)

b. (a)

$$r \dot{\theta} = u$$

$$r = a \sec^2 \theta/2 \Rightarrow \dot{r} = (2a \sec^2 \theta/2) \cdot \frac{1}{2} \cdot \sec^2 \theta/2 \tan \theta/2 \dot{\theta}$$

$$= a \sec^2 \theta/2 \tan \theta/2 \left( \frac{u}{a \sec^2 \theta/2} \right)$$

$$= u \tan \theta/2$$

Hence  $|\dot{r}| = |u \tan \theta/2|$  (\*)

BI  
MI AI  
MI  
AI (5)

(b)

$$\ddot{r} - r \dot{\theta}^2 = \frac{u \sec^2 \theta/2 \cdot \dot{\theta}}{2} - r \left( \frac{u}{r} \right)^2$$

$$= \frac{u}{2} \cdot \frac{r}{a} \cdot \frac{u}{r} - \frac{u^2}{r}$$

$$= \frac{u^2}{2a} (1 - 2 \cos^2 \theta/2) = -\frac{u^2 \cos \theta}{2a}$$

MI AI  
MI AI (4)

(c)

$$\frac{1}{r} \frac{d}{dt} (r^2 \dot{\theta}) = \frac{1}{r} \frac{d}{dt} (ur) = \frac{u}{r} \dot{r}$$

$$= \frac{u^2 \tan \frac{1}{2} \theta}{a \sec^2 \theta/2}$$

$$= \frac{u^2 \sin \theta/2 \cos \theta/2}{a} = \frac{u^2 \sin \theta}{2a}$$

MI  
AI (2)

(d)

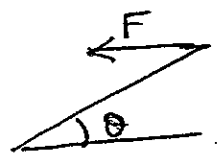
$$|F| = m \sqrt{\left( -\frac{u^2 \cos \theta}{2a} \right)^2 + \left( \frac{u^2 \sin \theta}{2a} \right)^2}$$

$$= \frac{m u^2}{2a}$$

Show indep<sup>t</sup> of  $\theta$

MI AI  
MI AI (4)

(e)



$\parallel^r$  to  $\theta=0$   
Dir<sup>2</sup>

MI  
AI (2)

(17)